4189. *Proposed by Mihaela Berindeanu.*

Prove that the equation

$$3y^2 = -2x^2 - 2z^2 + 5xy + 5yz - 4xz + 1$$

has infinitely many solutions in integers.

We received 15 solutions, most of which solved the given equation. We present the solution by Arkady Alt.

Rearrange and factor the given equation:

$$\begin{aligned}
&2x^2 + 4xz + 2z^2 + 3y^2 - 5xy - 5yz = 1 \Leftrightarrow \\
&2(x+z)^2 - 5y(x+z) + 3y^2 = 1 \Leftrightarrow \\
&(2(x+z) - 3y)(x+z-y) = 1.
\end{aligned}$$

Let $w = x + z$; if x, z are integers then so is w. To find the integer solutions to the equation $(2w - 3y)(w - y) = 1$ we consider two cases:

$$\begin{gathered}
2w - 3y = w - y = 1 \text{ OR} \\
2w - 3y = w - y = -1.
\end{gathered}$$

The first system of equations yields $w = 2$ and $y = 1$, and the second $w = -2$ and $y = -1$. From $x + y = w$, we get that the solutions to the original equation are $\{(t, 1, 2 - t) : t \in \mathbb{Z}\} \cup \{(t, -1, -2 - t) : t \in \mathbb{Z}\}$, showing that there are infinitely many solutions.

4190. *Proposed by Leonard Giugiuc.*

Let a, b, c, d and e be real numbers such that $a + b + c + d + e = 20$ and $a^2 + b^2 + c^2 + d^2 + e^2 = 100$. Prove that

$$625 \leq abcd + abce + abde + acde + bcde \leq 945.$$

Only the proposer supplied a solution.

Suppose, wolog, that $a \geq b \geq c \geq d \geq e$. We first verify that a, b, c, d, e must be all nonnegative. Since

$$\begin{aligned}
4(a^2 + b^2 + c^2 + d^2) &- (a + b + c + d)^2 \\
&= 3(a^2 + b^2 + c^2 + d^2) - 2(ab + ac + ad + bc + bd + be) \\
&= (a-b)^2 + (a-c)^2 + (a-d)^2 \\
&\quad + (b-c)^2 + (b-d)^2 + (c-d)^2 \geq 0,
\end{aligned}$$

then

$$(a + b + c + d)^2 \leq 4(a^2 + b^2 + c^2 + d^2) \leq 400.$$